Finite Element Models for Magnetic Shields made of Stacked Tapes

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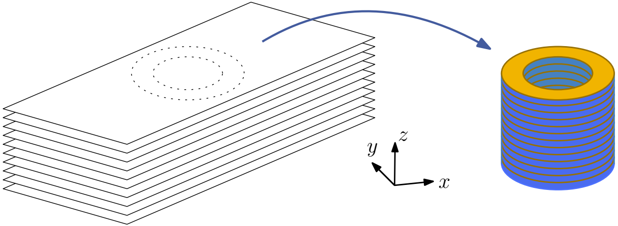
Magnetic Shield

The magnetic shield is made up of a stack of tape annuli.



Inner radius: 13 mm. Outer radius: 22.5 mm. Height: 14.9 mm.

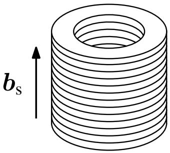
S. Hahn, 2011. A. Patel, 2016.

- Number of tapes: $N = 183$. One tape: HTS layer $+$ ferromagnetic (FM) substrate.
- Filling factor of the FM: $f = 0.92$.
- Temperature: 77K.

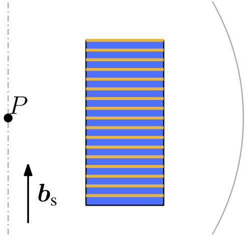
Shielding Factor (SF)

$$\text{SF} = \frac{\|\boldsymbol{b}_\text{s}\|}{\|\boldsymbol{b}(P)\|}$$

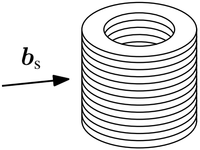
Axial



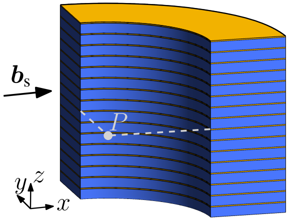
2D-axisymmetric



Transverse

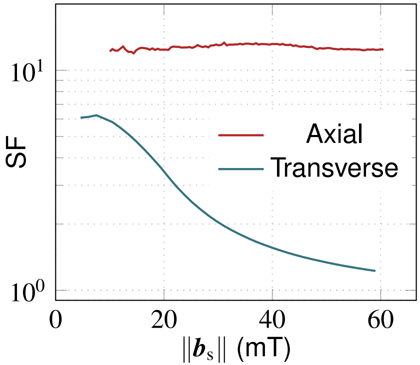


3D

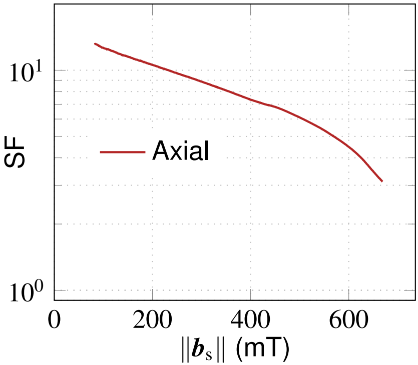


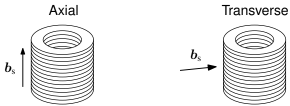
Experimental measurements (77K)

$\|\boldsymbol{b}_{s,\max}\| = 60$ mT, $\|\dot{\boldsymbol{b}}_s\| = 0.75$ mT/s.



$\|\boldsymbol{b}_{s,\max}\| = 670$ mT, $\|\dot{\boldsymbol{b}}_s\| = 5$ mT/s.





[See S. Brialmont's presentation tomorrow at 4:30 pm.]

Magneto-Quasistatic Equations

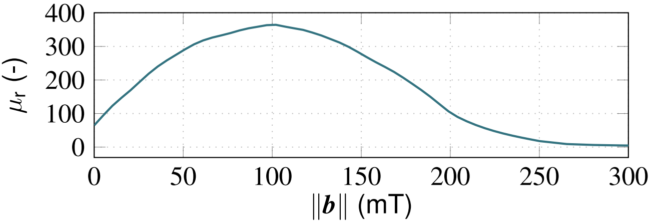
$$\boxed{\operatorname{div} \boldsymbol{b} = 0, \quad \textbf{curl}\, \boldsymbol{h} = \boldsymbol{j}, \quad \textbf{curl}\, \boldsymbol{e} = -\partial_t \boldsymbol{b}}$$

- HTS layer: μ_0, power law + Kim's model

$$\rho(\boldsymbol{j}) = \frac{e_c}{j_c(\boldsymbol{b})} \left(\frac{\|\boldsymbol{j}\|}{j_c(\boldsymbol{b})} \right)^{n-1} \quad \text{with} \quad j_c(\boldsymbol{b}) = \frac{j_{\mathrm{c0}}}{1 + \|\boldsymbol{b}\|/b_0},$$

  and n, j_{c0} and b_0 to be fixed (see later).

- FM substrate: $\mu_{\mathrm{r}}(\boldsymbol{b})$ from measurements at 77K, $\sigma = 0$.
  [See S. Brialmont's presentation tomorrow at 4:30 pm.]



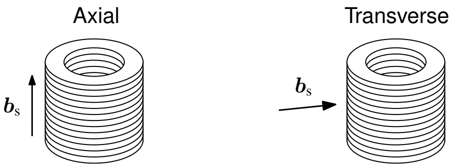
Two Models

Modelling $N = 183$ tapes is very expensive.

We propose two simplified models:

1. Simple model: $N_1 < N$ layers.
   - h-ϕ-b-formulation.
2. Homogeneous model: hybrid anisotropic material.
   - h-ϕ-formulation.
   - Accurate results with reasonable mesh resolution.

We run simulations in axial (2D-axi) and transverse (3D) cases.

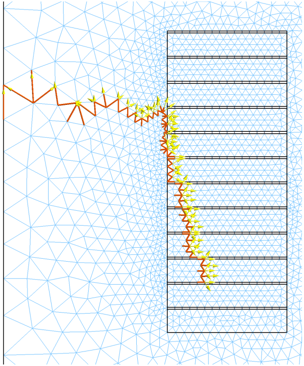


Simple Model (1/2): h-ϕ-formulation

Find $\boldsymbol{h}$ in the chosen function space ($\boldsymbol{h}$-ϕ) such that, $\forall \boldsymbol{h}'$,

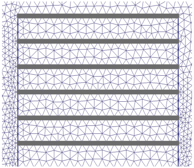
$$\left(\partial_t(\mu\,\boldsymbol{h})\ ,\boldsymbol{h}'\right)_{\Omega} + \left(\rho\,\mathbf{curl}\,\boldsymbol{h}\ ,\mathbf{curl}\,\boldsymbol{h}'\right)_{\Omega_\mathrm{c}} + \sum_{i=1}^{N_1} V_i \mathcal{I}_i(\boldsymbol{h}') = 0.$$

- $\Omega_\mathrm{c} \Rightarrow$ conducting domain (HTS).
- Edge functions ($\boldsymbol{h}$) in Ω_c,
  curl-free functions ($\mathbf{grad}\ \phi$) in $\Omega_\mathrm{c}^\mathrm{C}$.
- $V_i = 0, \forall i \Rightarrow$ no applied voltage (weak constraint).
- $\mathcal{I}_i$ is the net current in tape i (not imposed).
- Cohomology functions are used.



Simple Model (2/2): another formulation

In the h-ϕ-formulation, dealing with μ is not always robust. The nonlinearity is best solved as $\nu = \mu^{-1}$.



h-ϕ-a-formulation ?

- h-ϕ in Ω_c, a in $\Omega_\mathrm{c}^\mathrm{C}$ and surface coupling.
- Large coupling surface $\Rightarrow$ not optimal.

h-ϕ-b-formulation

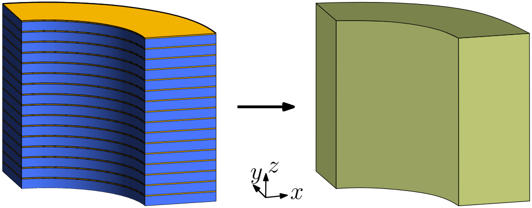
- h-ϕ in Ω and auxiliary $\boldsymbol{b}$ field in Ω_m (FM domain). Volume coupling in Ω_m:

$$\left(\partial_t \boldsymbol{b}\ ,\boldsymbol{h}'\right)_{\Omega_\mathrm{m}} + \left(\mu_0 \partial_t \boldsymbol{h}\ ,\boldsymbol{h}'\right)_{\Omega_\mathrm{m}^\mathrm{C}} + \left(\rho\, \textbf{curl}\, \boldsymbol{h}\ ,\textbf{curl}\, \boldsymbol{h}'\right)_{\Omega_\mathrm{c}} = 0$$
$$\left(\nu \boldsymbol{b}\ ,\boldsymbol{b}'\right)_{\Omega_\mathrm{m}} - \left(\boldsymbol{h}\ ,\boldsymbol{b}'\right)_{\Omega_\mathrm{m}} = 0$$

- If Ω_m is non-conducting, inf-sup condition satisfied with piecewise constant elements for $\boldsymbol{b}$.
- Much more robust than h-ϕ-formulation.

Homogeneous Model: Anisotropy

Replace the detailed stack by one homogeneous material.

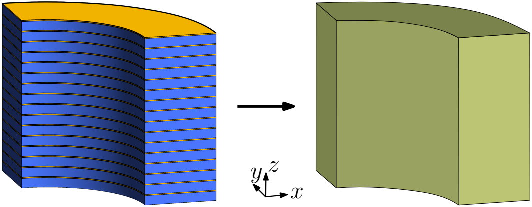


- Introduce the average $\boldsymbol{h}$ and $\boldsymbol{j}$ fields.
- Introduce anisotropic $\tilde{\boldsymbol{\rho}}(\boldsymbol{j})$ and $\tilde{\boldsymbol{\mu}}(\boldsymbol{h})$ tensors.
- Solve the h-ϕ-formulation:

$$\left(\partial_t(\tilde{\boldsymbol{\mu}}\,\boldsymbol{h})\ ,\boldsymbol{h}'\right)_{\Omega} + \left(\tilde{\boldsymbol{\rho}}\,\mathbf{curl}\,\boldsymbol{h}\ ,\mathbf{curl}\,\boldsymbol{h}'\right)_{\Omega_{\mathrm{c}}} + V_0\mathcal{I}_0(\boldsymbol{h}') = 0$$

with $V_0 = 0$.

Homogeneous Model: Permeability



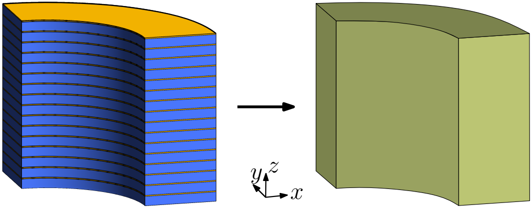
- Filling factor of FM: f.
- Field in the FM (implicit equation):

$$\boldsymbol{h}^{\mathrm{F}} = \begin{pmatrix} h_x^{\mathrm{F}} \\ h_y^{\mathrm{F}} \\ h_z^{\mathrm{F}} \end{pmatrix} = \begin{pmatrix} h_x \\ h_y \\ \mu_0 h_z / \left(f\mu_0 + (1-f)\mu(\boldsymbol{h}^{\mathrm{F}}) \right) \end{pmatrix}$$

- Permeability tensor:

$$\tilde{\boldsymbol{\mu}}(\boldsymbol{h}^{\mathrm{F}}) = \begin{pmatrix} f\mu(\boldsymbol{h}^{\mathrm{F}}) + (1-f)\mu_0 & 0 & 0 \\ 0 & f\mu(\boldsymbol{h}^{\mathrm{F}}) + (1-f)\mu_0 & 0 \\ 0 & 0 & \dfrac{1}{f/\mu(\boldsymbol{h}^{\mathrm{F}}) + (1-f)/\mu_0} \end{pmatrix}$$

Homogeneous Model: Resistivity



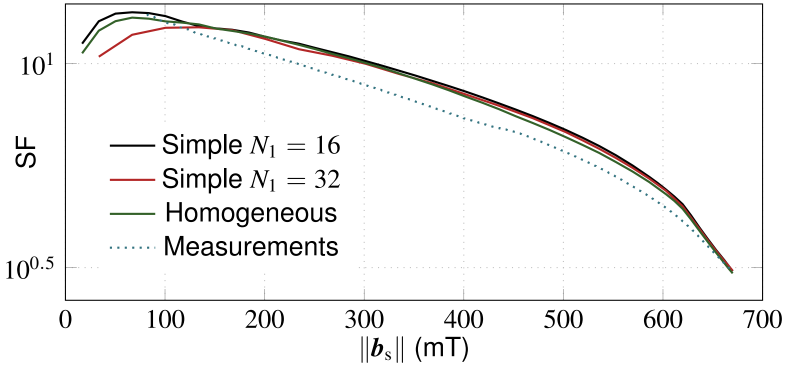
- Filling factor of HTS: $1-f$.
- Current in HTS: $\boldsymbol{j}^{\mathrm{S}} = \boldsymbol{j}/(1-f)$.
- Resistivity tensor, with ρ_∞ as large as possible ($0.01\ \Omega$m):

$$\tilde{\boldsymbol{\rho}}(\boldsymbol{j}^{\mathrm{S}}) = \begin{pmatrix} \rho(\boldsymbol{j}^{\mathrm{S}}) & 0 & 0 \\ 0 & \rho(\boldsymbol{j}^{\mathrm{S}}) & 0 \\ 0 & 0 & \rho_\infty \end{pmatrix}$$

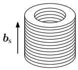
with $j_c = j_c(\boldsymbol{b}^{\mathrm{S}})$ (field in the HTS, not the average field).

Axial Case (2D-axi): model fit and verification

Good agreement with $n = 20$, $j_{c0} = 7.5 \times 10^9$ A/m^2, $b_0 = 0.1$ T.

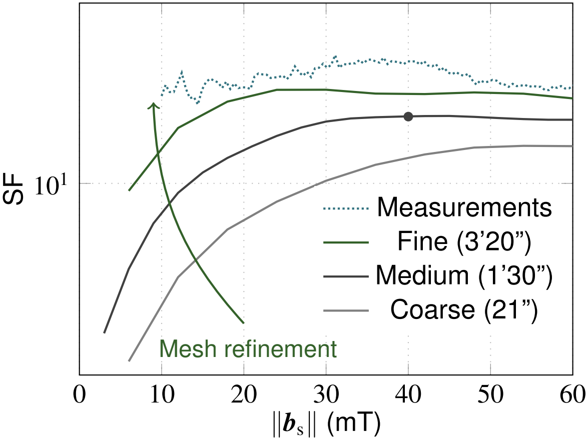


- Models converge towards the same solution with mesh refinement.
- The influence of N_1 is limited as soon as $N_1 \gtrsim 8$.
- All are sensitive to mesh resolution at low fields.

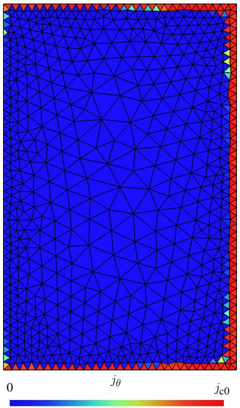


Axial Case (2D-axi) at Low Fields

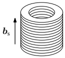
Homogeneous model at low fields. Different mesh resolutions.



Current density.



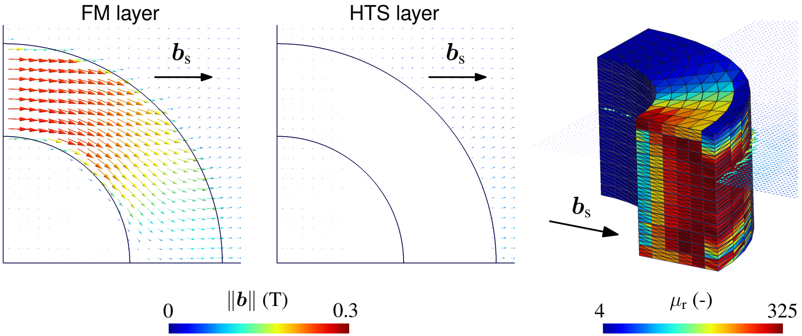
$\boldsymbol{b}_s = 40$ mT.
Medium mesh.



Same observation for the simple model.

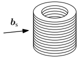
Transverse Case (3D)

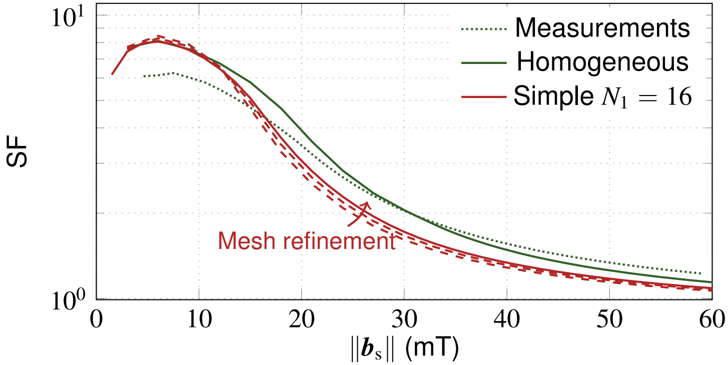
Solution at $\|\boldsymbol{b}_{\mathrm{s}}\| = 30$ mT. Simple model with structured mesh.



Transverse Case (3D)

Same observations, but the simple model is more expensive.





- At fine meshes, models are equivalent.
- The homogeneous model gives better results quicker.
- CPU time: simple ≈ 30 min .
- CPU time: homogeneous $\approx 10 - 15$ min .

Conclusion and Further Works

Conclusions:

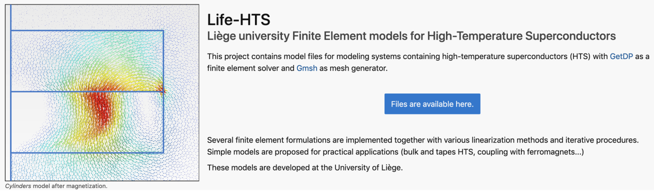
- Both models give good agreement with measurements.
- For the simple model, the h-ϕ-b-formulation is a robust option.
- The homogeneous model can be twice faster in 3D.

Future investigations:

- How to adapt the h-ϕ-b-formulation to conducting domains ?
- More clever simple model discretization?
- Do quadratic elements improve the model performance?
- How does a thin shell model compare to these models?

References

- GetDP, Gmsh.
- Life-HTS website: `http://www.life-hts.uliege.be/`



- *A stack of YBCO annuli, thin plate and bulk, for micro-NMR spectroscopy*, S. Hahn, et al. (IEEE TAS 2011).
- *Magnetic levitation using a stack of high temperature superconducting tape annuli*, A. Patel, et al. (SUST 2016).
- *Measurement of magnetic hysteresis loops of the Ni-5at.%W alloy substrate as a function of temperature in a stack of 2G coated conductor annuli*, S. Brialmont, et al. (in press, IEEE TAS 2022).
- *What Formulation Should One Choose for Modeling a 3D HTS Motor Pole with Ferromagnetic Materials?*, J. Dular, K. Berger, C. Geuzaine and B. Vanderheyden. (IEEE Trans. Magn. 2022).

Appendix

h-ϕ-a-formulation

- h-ϕ-formulation in Ω_c, a-formulation in Ω_c^C and coupling via common surface Γ_m:

$$\left(\partial_t(\mu \boldsymbol{h}) \ , \boldsymbol{h}'\right)_{\Omega_c} + \left(\rho \, \mathbf{curl}\, \boldsymbol{h} \ , \mathbf{curl}\, \boldsymbol{h}'\right)_{\Omega_c} + \left\langle \partial_t \boldsymbol{a} \times \boldsymbol{n}_{\Omega_c} \ , \boldsymbol{h}' \right\rangle_{\Gamma_m} = 0$$

$$\left(\nu \, \mathbf{curl}\, \boldsymbol{a} \ , \mathbf{curl}\, \boldsymbol{a}'\right)_{\Omega_c^C} - \left\langle \boldsymbol{h} \times \boldsymbol{n}_{\Omega_c^C} \ , \boldsymbol{a}' \right\rangle_{\Gamma_m} = 0$$

- For stability, second-order functions for $\boldsymbol{a}$ on Γ_m.
- Important coupling surface $\Rightarrow$ not optimal.