# Inverse analysis of critical current density in a bulk HTS undulator for field tuning

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***Abstract*—In order to obtain high-uniformity field in undulators using high-temperature superconductor (HTS) bulks, we have developed a method to estimate the critical current density (Jc) of each bulk from the measured magnetic field. The magnetic field was measured along the electron-beam axis in a HTS bulk-based undulator consisting of twenty bulks inserted in a 12-T solenoid. The Jc values of the bulks were estimated by an inverse analysis approach in which the magnetic field was calculated by the forward simulation of the shielding currents in each HTS bulk with a given Jc values as parameters. Subsequently the Jc values were iteratively updated using the pre-calculated response matrix of the undulator magnetic field to Jc values. We demonstrate that it is possible to determine the Jc of each HTS bulk with sufficient accuracy for practical application within around 10 iterations. The pre-calculated response matrix, created in advance, enables the inverse analysis to be performed within a practically short time, on the order of several hours.**

***Keywords— Inverse analysis of Jc, HTS application, Accelerators, Undulators, COMSOL Multiphysics, ANSYS***

## I. Introduction

HTS-based undulators have several advantages over conventional undulators employing either permanent magnets or low-temperature superconductors. HTS bulks can provide fields an order of magnitude stronger than those generated by permanent magnets. HTS materials have reduced cooling requirements and are thus more suitable than low-temperature superconductors in high current accelerators, in which the thermal input from the electron beam and the radiation is high. In the preliminary experiments of HTS bulk staggered array undulator (HSAU) conducted so far, the amplitude of the undulator magnetic field was as expected. However, the variation of the field amplitude among poles is not compatible yet with the practical usage as undulators because of the large individual differences of $J_c$ in the bulks. In this study we have developed a method to estimate the critical current density ($J_c$) of each bulk from the measured on-axis magnetic field.

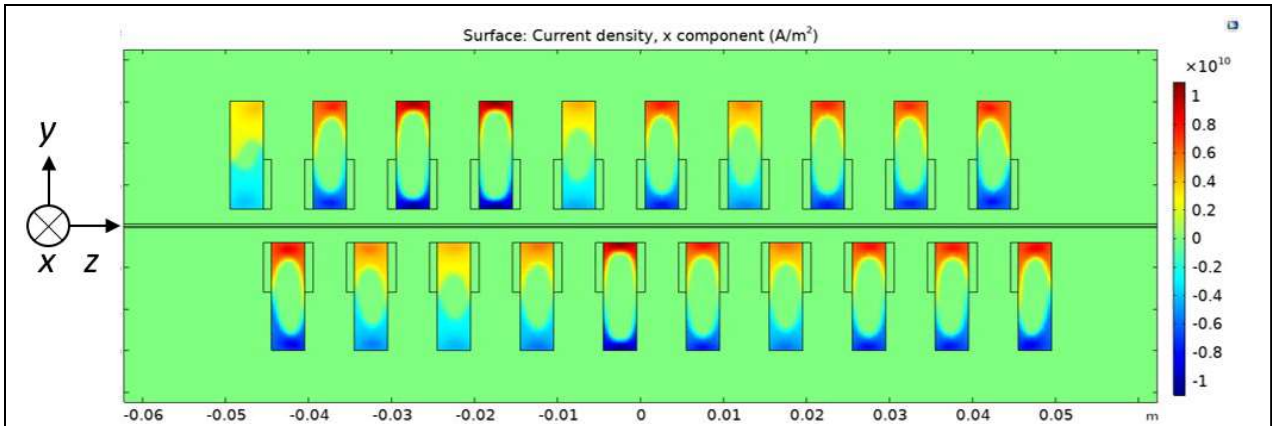


Fig. 1. Simulation geometry of HSAU and an example of the current density distribution ($J_x$) after the inverse analysis.

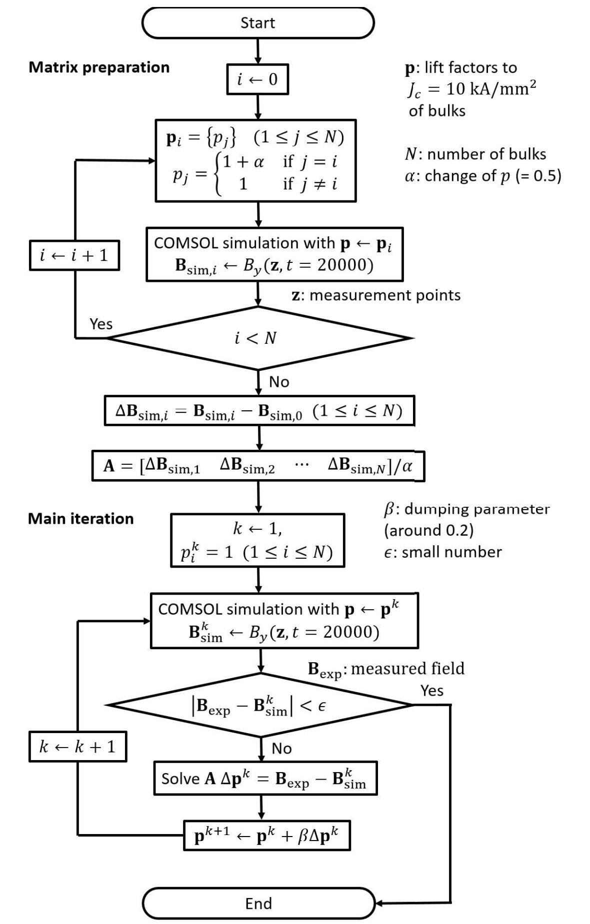


Fig. 2. Flowchart of inverse analysis by matrix method.

## II. METHOD

### A. *Experiment*

Measurements were performed in a 12-T solenoid at the University of Cambridge equipped with a large bore variable temperature insert. The 10-period prototype consisted by the 20 samples of HTS bulk material with half-moon shape are stacked in the staggered array configuration. The prototype was cooled down to 10 K with the solenoid field of 8 or 10 T and then the solenoid field was slowly ramped down to zero.

### B. *Inverse analysis by matrix method*

Fig. 1 shows the 2D simulation geometry of HSAU. The large 20 boxes indicate the bulks and the 38 small boxes indicates the ferromagnetic poles to enhance and adjust the on-axis field used in some experiments. Fig 2 shows the algorithm of the inverse analysis implemented by COMSOL Java functionality with Apache Common Math library. The vector **p** is the target of inverse analysis and, $p_j$ represents , $J_c$ of the $j$-th bulk ($1 \le j \le 20$) normalized by 10 kA/mm$^2$ (expected $J_c$ at 10 K and zero field). In the matrix preparation part, the response matrix **A** is made from the $B_y$ changes with changing one of **p**. In main iteration part, **p** is obtained by the iterative simulation by updating using the difference between the measured $B_y$ and the calculated $B_y$ in each iteration.

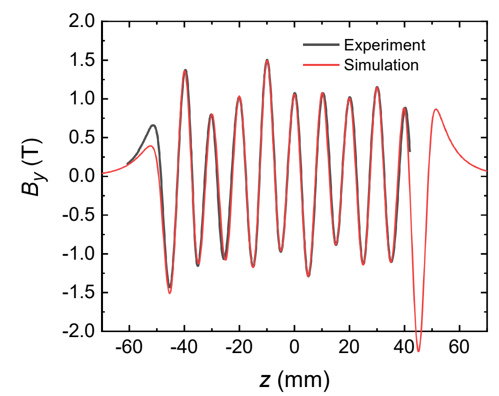


Fig. 2. An example of measured field profile (black) and reproduced field profile with estimated $J_c$ values (red).

### C. *Validation check*

The validity of the matrix method was checked by the inverse analysis with the given **p** by the random generator. Compared with the general purpose optimizer in COMSOL. The matrix method shows rapid conversion and high-accurary estimation at the same iteration number, and can estimate, **p** with sufficient accuracy (less than a few percent of standard deviation of given **p**) within 10 iterations even with the pre-calculated response matrix.

## III. RESULT AND DISCUSSION

Fig. 1. shows estimated current density distribution ($J_x$) in the HSAU after the inverse analysis. Fig. 2 shows the measured $B_y$ and calculated $B_y$with estimated $J_c$ values. The inverse analysis well reproduces the measured field.

For more detail, please see the paper about the inverse analysis [1]. In the workshop, we will also show that the virtual tuning by changing the order of HTS bulks (sorting) and by changing the position of bulks or poles (shimming) in the simulation can achieve the small variation field enough to practical applications. We will also show the result of inverse analysis recently done by the neural networks. The estimation of the $J_c$ is accurate as the matrix method and much faster than the matrix method.

## ACKNOWLEDGMENT

The inverse analysis in this presentation is described in detail in the paper "Inverse analysis of critical current density in a bulk HTS undulator" [1] by R. Kinjo, et. al. The work was done under the auspices of CHART (Swiss Accelerator Research and Technology Collaboration). M D Ainslie would like to acknowledge financial support from an Engineering and Physical Sciences Research Council (EPSRC) Early Career Fellowship, EP/P020313/1. The authors would like to thank Dr. Marek Bartkowiak from *Paul Scherrer Institute* for calibrating Hall sensors with PPMS.

## REFERENCES

[1] R. Kinjo, et. al., "Inverse analysis of critical current density in a bulk high-temperature superconducting undulator", Phys. Rev. Accel. Beams, 25, 043502 (2022).