Fast integral modeling approach of large-scale non-inductive HTS coils

Sara Fawaz, Hocine Menana and Bruno Douine
Group of Research in Electrical Engineering of Nancy (GREEN)
University of Lorraine, Nancy F-54000, France
email address: (name.surname@univ-lorraine.fr)

***Abstract*—In this paper, a fast integral modeling approach is proposed for the design and the characterization of large-scale non-inductive high temperature superconducting coils. The calculated AC losses are compared to that obtained with Norris' formulas and to measurements.**

***Keywords*—AC losses, Large-scale non-inductive HTS coils, Measurements, Volume integral equations.**

I. Introduction

High temperature superconducting coils present a great interest for electric power applications, such as electrical machines and superconducting power filters. For an accurate design of such coils, numerical models are required to predict the electromagnetic properties consisting of the critical operating current and the AC dissipated losses.

Several approaches have been developed for the modeling of HTS, the finite element method based on H formulation is the most commonly used. However, when the modeled system consists of a set of distributed sources and has multi-scale dimensions, such as HTS coils, this method becomes costly in memory space and computation time. In this case, integral methods are more suitable because it limits the discretization to the active parts of the system and no boundary conditions are required [1-2]. When the coils are of large size (great number of turns), even the integral approaches present difficulties to model such systems, due to the great number of the discretization elements. In this case, modeling strategies have to be adopted to reduce the size of the matrices and the computing time, such as the fast multipole method [3] leading to a far-tape approximation strategy in this case [4]. In this work, the far-tape approximation is applied to model a large scale non-inductive HTS coil carrying AC currents, leading to a successive modeling of a very reduced set of tapes of the HTS coil. This results in a considerable gain in memory space and computing time.

II. The Modeling Approach

The modeled system consists of an N_t-turn circular pancake coil of inner and outer radii, respectively denoted R_{int} and R_{out}, wound with a first generation HTS tape of width H_t and thickness W_t as illustrated in Fig. 1. The electrical behavior of the HTS materials is modeled by an *E-J* power law, associated with Kim's law to consider the $J_c(B)$ dependency, both given in (1), where E and E_c are the electric field and its critical value; n is the creep exponent; J_{c0} is the critical current density at the zero magnetic field; B_0 and β are physical parameters depending on the considered superconducting material; and k is a parameter of anisotropy.

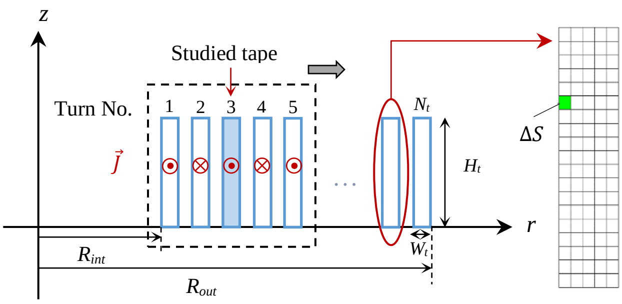

Fig. 1. The cross-section geometry of the HTS coil and its discretization.

$$\begin{cases} E(J,B) = E_c\left|J\,J_c^{-1}(B)\right|^n \\ J_c(B) = J_{c0}\left(1 + B_0^{-1}\sqrt{k^2 B_z^2 + B_r^2}\right)^{-\beta} \end{cases} \quad (1)$$

Since the coil is non-inductive, and the current is imposed in the coil, the far-tape approximation allows us to transform the hole coil modeling to a successive modeling of an odd set (n_t) of consecutive tapes, where the physical quantity of interest is evaluated in the middle tape. The sequence is repeated to deal with all the coil tapes by sliding the n_t consecutive tapes by one tape at each iteration. The discretization leads thus to only a number ($N_e = n_t \times N_r \times N_z$) of elementary sections (ΔS) for each computation sequence. The distributions of the magnetic vector potential $\bar{A}$, and both of the radial and axial components ($\overline{B_r}, \overline{B_z}$) of the MFD in the tapes section are given in (2), where $\overline{\overline{G_A}}$, $\overline{\overline{G_{BR}}}$ and $\overline{\overline{G_{BZ}}}$ are ($N_e \times N_e$) integral matrices and $\bar{J}$ is a vector containing the current densities of the N_e elementary sections.

$$\left\{\bar{A} = \overline{\overline{G_A}}\,\bar{J};\quad \overline{B_r} = \overline{\overline{G_{BR}}}\,\bar{J};\quad \overline{B_z} = \overline{\overline{G_{BZ}}}\,\bar{J}\right\} \quad (2)$$

The AC modeling approach is based on the expression of the electric field as function of the magnetic vector and electric scalar potentials ($\vec{A}, V$) given by (3), considering the non-linear *E(J)* characteristic.

$$\partial_t \vec{A} + E_c\left|\frac{J}{J_c}\right|^n \frac{\vec{J}}{\|\vec{J}\|} + \vec{\nabla} V = \vec{0} \quad (3)$$

The value of $\vec{\nabla} V$ is constant over the entire cross section of a tape, and can be used to impose the conservation of the current in each tape [5]. The matrix form of (3) can thus be expressed as follows:

$$\overline{\overline{G_A}}\,\partial_t \bar{J} = -E_c\{\bar{J}/\bar{J_c}\}^n - \gamma\overline{\overline{S}}\{\bar{J} - \bar{J_s}\} \quad (4)$$

where γ is a feedback constant, and $\bar{J_s}$ is applied in such a way that the current flows in the opposite direction in two consecutive turns. The matrix system (4) is solved using the ODE solver "ode15s" in Matlab.

III. Results and Discussions

An insulated non-inductive HTS coil made of BSCCO/Ag multi-filamentary tape has been constructed and characterized. The parameter specifications are given in Table I. The measured critical current of the coil is 168 A, while that of the tape is 170 A, highlighting the effect of the geometry on the HTS performances.

TABLE I. Parameters Specifications

Parameter	***Value***	***Description***
I_c *(77K)*	170 *A*	Tape critical current at zero external field
L	102 *m*	Length of the superconducting tape
W_t / H_t	0.23/4.3 *mm*	Tape thickness / Tape width
R_{out}/R_{int}	27 / 9 *cm*	Outer/Inner radius of the coil
N_t	92	Number of the coil turns
E_c	1 *µV/cm*	Critical electric field
n	15	Creep exponent
I_{c0}	184.51 *A*	Critical current at zero MFD
k	0.14	Parameter of anisotropy
B_0/β	0.14 T /2.25	Constants used in (1)
γ	100 *V/(A.m)*	Feedback constant

The developed approach is applied to the HTS coil for the determination of the current density *J* distribution and the evolution of the AC losses dissipated for each value of the applied current.

Figure 2 shows the distribution of *J* in the last sequence (last five tapes), for a sinusoidal applied current of 115 A amplitude and 40 Hz frequency, at three different instants: t=T/4, t=T/2, and t=3T/4, where T is the period. In the plots, the thickness of the superconducting tapes has been expanded in the r-direction to better highlight the current repartition. Since the coil is non-inductive, we can notice that the current flows in the opposite direction in two consecutive turns and penetrates each tape from the outside, until a distance depending on the critical current. When the current decrease, an inverse current is induced from the outside, so that when the applied current is zero at t=T/2, the current density in each tape is not locally nil (Fig. 2-b), but its integral over the surface of each tape is nil. At t=3T/4, we have a reverse situation compared to t=T/4.

The AC losses dissipated in the HTS coil for a frequency of 40 Hz, using the modeling approach are reported in Fig. 3, compared to the measurements and the losses calculated with Norris formulas at the same frequency. As we can notice, the experimental and numerical results show a good agreement and they are located between the Norris's formulas given for a strip and elliptical section.

References

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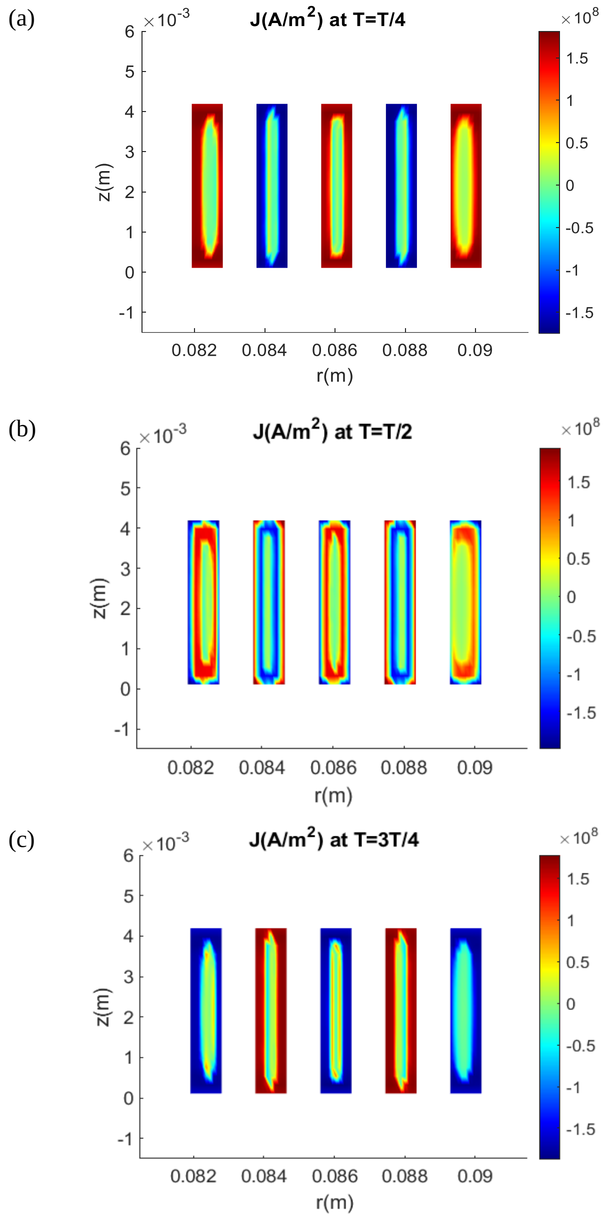

Fig. 2. Distribution of the electric current density in the last five tapes cross section at 40 Hz, for an applied current $i(t) = 115\sin(2\pi f t)$ with: (a) t=T/4, (b) t=T/2, (c) t=3T/4.

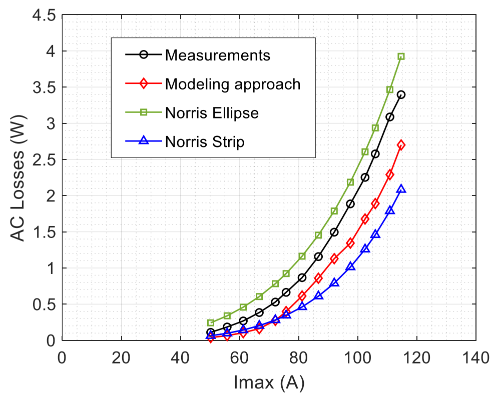

Fig. 3. Comparison between the numerical and experimental AC losses and the losses calculated with Norris formulas for a frequency of 40 Hz.