# Novel results obtained by modeling of dynamic processes in superconductors

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***Abstract* — Based on the time-dependent Ginzburg-Landau system of equations and finite element modeling we present novel results related with physics of phase-slippage in superconducting wires surrounded by a non-superconductive environment. These results continue our previously reported modeling approach related to superconducting rings and superconductive gravitational wave detector transducers. It is shown that the phase-slip centers (PSCs) can be effective in originating both positive and negative thermal fluxes. With an appropriate design utilizing this nonequilibrium physics, cooling below 1K is possible to achieve.**

***Keywords—TDGL equations, finite element modeling, phase-slip centers, negative phonon fluxes, cryocooling***

## I. INTRODUCTION

During the last decade the system of time-dependent Ginzburg-Landau (TDGL) equations in parallel with COMSOL Multiphysics finite element modeling were successfully used for studying various physical systems [1-4]. Using this approach, our group reported on violation of flux conservation theorem in superconductive nanorings [5], as well as on possibility of effective superconducting transducers for gravitational wave detectors [6]. The last two works targeted 3-D ring structures by extending the method described in [7] to superconductors with the finite gap value, in particular, to NbN superconductors. In these reports as well as in those considering dissipative resistive states related to PSCs, the phonon subsystem is considered equilibrium. In practice, this means that nonequilibrium phonons are freely leaving the superconductor without significant feedback. This can be justified by the fact that for thin enough (1-D) wires the phonon escape time is short enough. However, such a description may have at least two shortcomings. If the acoustic density of surrounding material is much lower than that of the superconductor, the Kapitza resistance [8] between superconductor and its surrounding greatly increases the role of nonequilibrium phonons. Inclusion of this factor in the modeling can deliver much better agreement with experimental findings. The second shortcoming is related with the neglecting of nonequilibrium phonon fluxes which themselves may have useful practical features. The latter aspect is the major topic of current report.

## I. COUPLED ELECTRON-PHONON DYNAMICS

We will consider a DC-biased superconducting fiber constricted between two massive superconducting banks. The diameter of this fiber is assumed to be smaller than the characteristic lengths of parameters' variation in nonequilibrium superconductor, such as the coherence, London penetration and electric field penetration lengths, $\xi(T)$, $\lambda_L(T)$, and $l_E(T)$ correspondingly, so as the 1D-model is applicable to the fiber. In this case, we can have two additional constraints to the one imposed by the gauge invariance, and thus we can use these constraints to eliminate the vector potential from the modeling equations (this approach is common at the description of PCSs). The remaining scalar potential $\varphi$, as well as the phase $\theta$ of the Cooper-pairs wave function $\Psi=|\Psi|\exp(\theta)$ are becoming then gauge invariant. The current density j equals to the total current value J up to the filament cross section S: j=J/S and will serve as an external parameter controlling nonequilibrium physics in the filament. In particular, at certain values $j>j_0$, the oscillatory regime sets up in the filament. Physical quantities, such as normal current $j_n$ and supercurrent $j_s$ oscillate in time, together with oscillations of the Cooper-pair density, which periodically reaches zero-value at some "weak" point of the filament (for symmetric arrangement, this point coincides with the filament center). At the moment of zeroing the modulus of the $\Psi$-function, its phase $\theta$ suffers a drop (i.e. slips) by $2\pi$ or its multiple. Fundamental physics of phase slipping was the matter of numerous studies for decades. Not only the physics of electronic system has been described in detail (see, e.g., monographs [7-11] and references therein), but also the phonon fluxes were predicted to change their sign periodically in time: emission is followed by absorption [12]. What was not revealed, however, is that at some choice of physical parameters of the filament and its surrounding environment, a mechanism of cryocooling becomes possible. This is the major topic of our report. To understand it, we should describe the oscillatory regime of the current flow and the coupled phonon dynamics in greater detail.

## II. NONEQUILIBRIUM ELECTRONS AND COOPER PAIRS

Using TDGL equations for finite gap superconductors (see, e.g., [7,9]) one can obtain the following physical picture of the Cooper-pair condensate behavior in terms of its $\Psi$-function, Fig. 1. The most important factor for our consideration is that $\partial|\Psi|/\partial\tau$, as follows from Fig. 1(b), is periodically changing its sign: it is negative almost during the whole period of $|\Psi|$-oscillation, and for a short while acquires gigantic positive values immediately after the moment of slippage. One can easily conclude that because of the charge conservation, the reduction of Cooper-pair density should be accompanied by an increase of single-electron population. Quantitatively, this can be described in terms of the Eliashberg-Keldysh energy-integrated Green's function formalism for nonequilibrium superconductivity. From this description it follows that the $|\Psi|$-value is coupled with the electron-hole excitation functions $n_\varepsilon$ and $n_{-\varepsilon}$ by the relation: $\delta(n_\varepsilon+n_{-\varepsilon}) \propto [-\partial|\Psi|/\partial\tau]$ and can be quantified.

## III. COUPLED PHONON DYNAMICS

The outlined fact has interesting consequences in the phonon system coupled with the nonequilibrium electrons. Most noticeably, the sign of phonon fluxes oscillates in time.

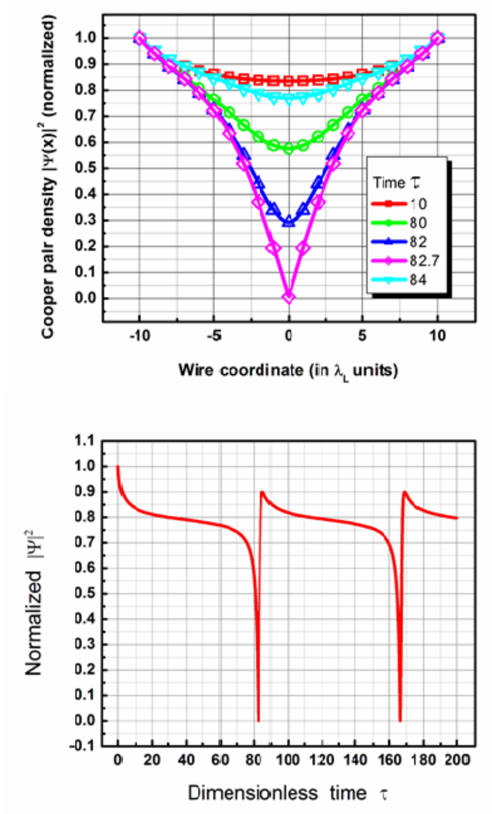

Fig. 1. (*top*) Spatial distribution of Cooper-pair density during different moments of time at oscillatory regime of phase-slippage ($j>j_0$). (*bottom*) Temporal behavior of $|\Psi|$ at the center of filament ($x=0$) starting from the equilibrium value ($|\Psi|=1$) at switching on $j>j_0$ at $\tau=0$.

Indeed, from the kinetic equations for the coupled electron-phonon system it can be deduced that phonons should be emitted when $\delta(n_\varepsilon+n_{-\varepsilon})>0$ and absorbed when $\delta(n_\varepsilon+n_{-\varepsilon})<0$. In average, the externally applied DC current dissipates power since occurrence of PSCs is associated with a resistive state. This resistance is smaller than in the normal state, but its time-average value is non-zero. This means that the positive phonon flux (i.e. the emitted phonon energy) is larger than its negative counterpart (i.e. the absorbed phonon energy). However, they are quite comparable, which creates opportunity of effective cooler based on this effect.

## IV. COOLING MECHANISM

There is one step from the effect of the phonon flux sign reversal to the concept of an all-solid-state cooler. Suppose that the filament is deposited on a substrate with higher acoustic density ρu, where ρ is the mechanical density, and u is the phonon propagation speed (the sound velocity). In this case phonons will be admitted by the substrate without total internal reflection. Contrary, if another solid plate is fused on the top of the filament with the value of ρu smaller than that of the superconductor (Fig. 2), the total internal reflection will partially restrict propagation of phonons released by the filament into it (the phenomenon of the Kapitza resistance takes place). Reciprocally, when the filament is in the phonon-absorption state, the thermal phonons from the acoustically less dense substance (top plate) will propagate into the filament without any restriction, while those from the substrate will suffer the total internal reflection.

## V. CONCLUSION

Thus, one can conclude that there is an opportunity to choose the parameters of the system in such a way that the outflow of phonons from the top plate is more effective than the inflow to it. We provide detailed modeling results confirming this conclusion. The asymmetry will result in cooling of the top-plate, while the substrate will stay at cold finger temperature. This opens principal opportunity of designing novel all-solid-state cryocooler.

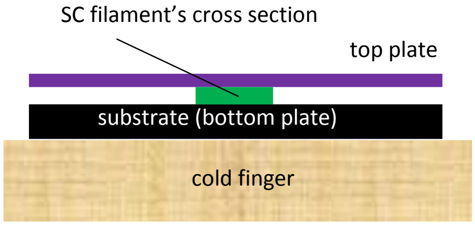

Fig. 2. Cross-sectional view of the cooler design.

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