3D Modelling of the Currents Induced in a Superconducting Linear Halbach array

Michel Houbart
Montefiore research unit
University of Liege
Liege, Belgium
michel.houbart@uliege.be

Jean-François Fagnard
Montefiore research unit
University of Liege
Liege, Belgium
JF.Fagnard@uliege.be

Julien Dular
Montefiore research unit
University of Liege
Liege, Belgium
julien.dular@uliege.be

Anthony Dennis
Bulk Superconductivity Group
University of Cambridge
Cambridge, United Kingdom
ad466@cam.ac.uk

Devendra Kumar Namburi
Quantum Sensors Group
University of Glasgow
Glasgow, United Kingdom
dkn23@cam.ac.uk

John Durrell
Bulk Superconductivity Group
University of Cambridge
Cambridge, United Kingdom
jhd25@cam.ac.uk

Christophe Geuzaine
Montefiore research unit
University of Liege
Liege, Belgium
cgeuzaine@uliege.be

Benoît Vanderheyden
Montefiore research unit
University of Liege
Liege, Belgium
B.Vanderheyden@uliege.be

Philippe Vanderbemden
Montefiore research unit
University of Liege
Liege, Belgium
Philippe.Vanderbemden@uliege.be

***Abstract*—The combination of permanent magnets in a so-called "Halbach array" is a commonly used method to generate large magnetic flux density gradients with a compact system and is of great interest in applications requiring a magnetic force. The maximum magnetic force achievable with such systems is, however, limited by the saturation magnetization of the permanent magnets used. A superconducting Halbach array could be an attractive alternative generating larger field gradients as trapped field superconducting magnets do not suffer from an intrinsic saturation of their magnetization. In this context, a 3D finite-element model is developed in the *GetDP* environment in order to investigate numerically the current density distribution in three pre-magnetized cubic superconductors assembled in a Halbach array configuration. The simulation shows that during the assembly of the array, the approaching peripheral samples induce a reorganisation of the current density distribution in the central one. This modification results in a reduction of the magnetic flux density generated above the centre of the configuration, which is expected to limit the performance of the system. The finite-element predictions are confronted both to analytical calculations and to experimental results, which confirm this picture.**

***Keywords*—Bulk superconductor, trapped field magnet, magnetic field gradient, interacting bulk superconductors, flux pinning**

I. Introduction

A commonly used method achieving large magnetic forces with a compact system consists in combining several permanent magnets with non-parallel magnetization directions in a so-called "Halbach array". The magnitude and the range of the magnetic force achievable with a conventional Halbach array is, however, limited by the saturation magnetization $\mu_0 M_{\text{sat}}$ of the magnetic material used. A promising alternative for permanent magnets which would address this limitation are the so-called "trapped-field" bulk superconducting magnets as their trapped field can be enhanced either by reducing the temperature or by increasing their dimensions [1], [2]. Assembling a superconducting Halbach array is, however, more critical than a conventional one because the current density trapped in each magnetized superconductor may potentially be altered by the proximity of other samples [3], [4]. Understanding and predicting these current density modifications is thus of prior importance in order to assess the performance of superconducting Halbach arrays.

In this context, a 3D finite-element model based on the "$\vec{A}$-formulation" developed in [5] in the *GetDP* environment is extended to investigate the interaction between several magnetized bulk superconductors in relative motion with respect to each other. The zero-field cooling magnetization of an individual cubic superconductor is first simulated to compute the initial $\vec{A}$ field distribution in each sample. Starting from this distribution, the position of the superconducting regions is then modified at each time step and the $\vec{A}$ field computed at the previous time step is projected in these regions in order to model the motion. Although this method requires a remeshing of the domain at each time step, which prevents us from using very fine meshes, the model gives an excellent qualitative understanding of the current distribution inside the superconductors. This can greatly help in developing analytical models and finally obtain a finer description of the $\vec{B}$-field generated by the superconducting Halbach array.

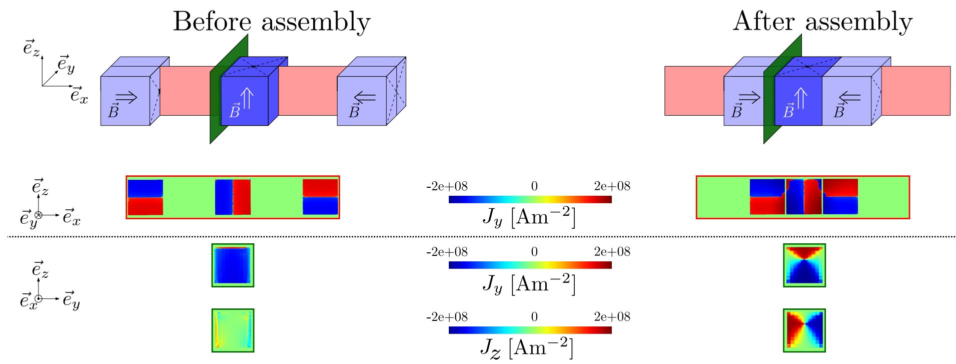

Fig. 1. Comparison of the y- and z-components of the current density distribution before and after the assembly of the superconducting Halbach array computed with the finite-element model. The results are presented in two different cut planes represented in red and green respectively.

II. Results

We first examine the current density inside the bulk superconductor, as determined by the 3D finite-element model. Fig. 1 compares the current density distributions before and after the assembly process of the superconducting Halbach array. The finite-element simulation predicts a reorganisation of the current density distribution inside the samples during the assembly process in a region close to the contact surface between neighbouring samples. This redistribution has two consequences: (i) a decrease in the contribution of the central sample to the magnetic flux density above the centre of the array, and (ii) the apparition of a new negative contribution to the magnetic flux density at this location.

Next, a simple analytical model is developed on the basis of the Biot-Savart law and on the assumption that the individual magnetizations are not altered during the assembly process [4]. The model is confronted to measurements performed at 77 K on an actual superconducting Halbach array in Fig. 2. The model assumes completely magnetized superconductors in the critical state (Bean model [6]) with square current loops strictly perpendicular to the magnetization direction. The critical current density is also assumed to be constant and field-independent over the whole superconductor. If the currents in the central sample are assumed to be unaltered by the approach of the two external samples, the model significantly overestimates the magnetic flux density generated above the centre of the configuration. Knowing, from the 3D modelling, that a redistribution of currents occurs in the central sample, a modified analytical model including the current density distribution modifications predicted by the finite-element simulation is also confronted to experimental data in Fig. 2. A satisfying agreement is obtained with the modified analytical model, which gives confidence in the qualitative description of the current distribution obtained with the finite-element simulation.

In summary, a 3D finite-element model was used to investigate the assembly of a superconducting Halbach array made of 3 bulk superconductors. The simulations showed that the peripheral sample approach causes a modification of the current density distribution in the central superconductor. The knowledge of this alteration allows one to develop a simple analytical model successfully capturing the main experimental features of the trapped $\vec{B}$ field.

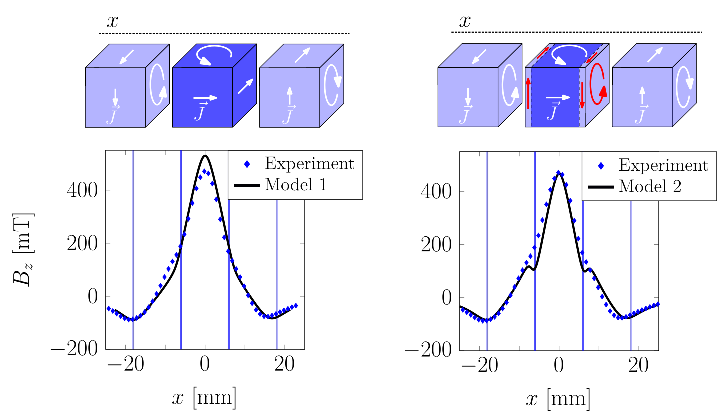

Fig. 2. Evolution of the z-component of the magnetic flux density generated 1 mm away from the surface of a Halbach array made up of three cuboid bulk superconductors along the line x. The analytical model considers a simple vector summation of the flux densities generated by each superconductors in the array, square current loops strictly perpendicular to the magnetization direction and a constant and field-independent critical current density. Model 1 assumes no alteration of initial current density distribution. Model 2 includes the modifications predicted by the finite-element simulation.

Acknowledgment

This work was supported by the Fonds de la Recherche Scientifique - FNRS under grant CDR n° J.0218.20 (35325237). Michel Houbart and Julien Dular are recipients of a FRS-FNRS Research Fellow grant.

References

[1] S Nariki, H Teshima, and M Morita 2016 *Supercond. Sci. Technol.* **29** 034002
[2] M Tomita, M Murakami 2003 *Letters to Nature* **421** 517–520
[3] M Houbart, J-F Fagnard, A R Dennis, D K Namburi, Y Shi, J H Durrell, and P Vanderbemden 2020 *Supercond. Sci. Technol.* **33** 064003
[4] M Houbart, J-F Fagnard, J Dular, A R Dennis, D K Namburi, J H Durrell, C Geuzaine, B Vanderheyden, and P Vanderbemden *Submitted to Supercond. Sci. Technol. in 2022*
[5] J Dular, C Geuzaine, and B Vanderheyden 2020 *IEEE Trans. Appl. Supercond.* **30** 8200113
[6] CP Bean 1962 *Phys. Rev. Lett.* **8** 250–253