On Static H-formulation Adapted to the Critical Current Model of HTS Conductors

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***Abstract*—A new attempt in H-formulation is presented, as an alternative to the static formulation of magnetic vector potential, to be applied to the self-consistent critical current model of HTS (High Temperature Superconducting) devices. Avoiding the intrinsic difficulty of the illposedness in static condition, the H-formulation is carefully implemented by critical combination of the known ideas which includes the BEM (Boundary Element Method) of the Laplace equation. On the merit of the vectorial field model, the efficacy of the alternative H-formulation is discussed not only in terms of optimal size of the air mesh, but also about modeling the background field, as improved to be easier than the previous work.**

Keywords—Critical Current, Numerical Simulations, H-formulation, Superconducting Cables

I. Introduction

The self-consistent critical current model has been recognized for its potential use to inspect the design of HTS (High Temperature Superconducting) devices by evaluating their current capability in terms of critical current, as typical in recent works, of the 2G REBCO (2nd Generation Rare Earth Barium Copper Oxide) coated tapes [1-2]. Proposed to be an agile method of 2-d simplification, the idea has been founded upon the condition of static current, as equivalent to the uniform electric field (*E*-field) over the cross section or each slice of the cross sectional planes [1].

Yielding rather efficient evaluations, a sensible trick of *P*-value indicator, which represents the domain-wise *E*-field uniformity, has been introduced not to solve the nonlinear *E*-*J* relationship directly, but to lead to the steady state solution restricted by $J_c(\vec{B})$ and the domain-wise uniformity P [1]. Under the idea, a numerically sound magnetic formulation should be laid, as the steady state transport is understood from the asymptotic behavior of magnetic induction. Thus, the equation of magnetic vector potential $\vec{A}$ was favorably chosen for the underlaid magnetic formulation, because its static limit guarantees a well-posed problem by itself.

In this study, an alternative approach based on the field strength $\vec{H}$ is attempted, instead of the magnetic vector potential, to substitute to the previous solution scheme based on the *P*-value. Taking advantage of direct access to the field quantity of interest, the alternative H-formulation is motivated to investigate the expected capabilities.

II. Model Description

In order to carry out the attempt, a particular variant of H-formulation has to be implemented to avoid the intrinsic difficulty of ill-posed problem, which appears in the static limit of H-formulation. In this sense, a legacy idea of penalization is invoked to prepare the H-formulation in weak form [3-4]. Substituting the penalized magnetic vector potential $\vec{A} = \frac{1}{\nu}(\vec{J} - \vec{\nabla} \times \vec{H})$, the weak form is closed on the domain of interest Ω, as written in terms of the field strength $\vec{H}$, the current density $\vec{J}$, and the arbitrary test field $\vec{H}'$ [3]. Besides, the boundary integral along $\partial\Omega$ is described by the reduced vector potentials, i.e., a and a_0, as defined for the background field $\vec{H}_0$ to be $\vec{\nabla} \times (a_0 \hat{z})$, and the H-field on the boundary to be $\vec{\nabla} \times (a\,\hat{z}) + \vec{H}_0$.

$$\begin{cases} \int\limits_{\Omega} \vec{H} \cdot \vec{H}' dx^2 + \frac{1}{\mu_0 \nu} \int\limits_{\Omega} \left(\vec{\nabla} \times \vec{H} - \vec{J} \right) \cdot \left(\vec{\nabla} \times \vec{H}' \right)\, dx^2 \\ \qquad + \oint\limits_{\partial\Omega} \left(a + a_0 \right) \vec{H}' \cdot \hat{\tau}\; dl = 0 \\ \oint\limits_{\partial\Omega} \frac{\partial a}{\partial n} a'\; dl + \oint\limits_{\partial\Omega} (\vec{H} \cdot \hat{\tau} - \vec{H}_0 \cdot \hat{\tau}) a'\; dl = 0 \end{cases} \tag{1}$$

To complete the formulation, the second equation of simple identity emerges, which leads the free boundary condition constrained by the stray field in the outside of Ω, virtually solving the Laplace equation of the reduced potentials through its Green's function kernels [4-5].

III. Numerical Model

Following the formulation in Eq. 1, the numerical model of magnetic field is composed on the FEM (Finite Element Method) model of curl-free (Nédélec's) vectorial shape function [6], and the BEM (Boundary Element Method) of simple linear shape function [5]. By virtue of the recent development of high-level modeling tool, the mathematical formulations are synthesized by the FreeFEM++'s lexicons in a straightforward manner [7], and composed to the matrix formula (Eq. 2) by discretization, as codes in simple idioms, in terms of the unknowns, i.e., $\mathbf{h}$ of the H-field and $\mathbf{a} + \mathbf{a}_0$ of the reduced vector potential on the boundary,

$$\begin{bmatrix} [\mathbf{F}] & [\mathbf{C}]^{\mathrm{T}} \\ [\mathbf{C}] & [\mathbf{B}] \end{bmatrix} \begin{bmatrix} \mathbf{h} \\ \mathbf{a} + \mathbf{a_0} \end{bmatrix} = \begin{bmatrix} [\mathbf{V}] \cdot \mathbf{j}(\mathbf{h}) \\ 2[\mathbf{C}] \cdot \mathbf{h_0} \end{bmatrix} \tag{2}$$

where the current density $\mathbf{j}$ and the external magnetic field $\mathbf{h}_0$ are given to the load vector on the right hand side of Eq. 2. The matrices are obtained from the numerical integrations over the shape functions. Thus, $[\mathbf{F}]$ and $[\mathbf{V}]$ are matrices induced from the integrals on the domain Ω, and $[\mathbf{C}]$ and $[\mathbf{C}]^{\mathrm{T}}$ are from the boundary integrals along $\partial\Omega$, coupling the vectorial (domain) elements and the boundary elements. Applying the Green's function formulae of the Laplace equation, $[\mathbf{B}]$ is prepared as a dense matrix of so called BEM discretization [5].

Through the final equation (Eq. 2) of block matrix, one is able to evaluate the nonlinear solution of $\mathbf{j}(\mathbf{h})$ from the *P*-value based solution scheme, to be absolutely parallel to the previous work. Thus, the FreeFEM++ code is written as an iterative routine to make the domain wise *P*-value indicators converged [2].

IV. Benchmark Result

As a benchmark test, the Roebel cable model of critical current is brought, which was introduced as the primary case in the reference [1-2]. For a better comparison, the same mesh structure of 10 HTS strands is applied to the alternative model of H-formulation [2], while the outer diameter of air mesh is 13 times smaller, thanks to the BEM technique [4-5], than the original A-formulation. In spite of such downsizing in diameter, the total number of DoF is rather increased, but just becomes comparable to the reference of A-formulation at the application of the vectorial element (the Nédélec's element) [6]; in detail, the A-formulation model has 6516 vertices and 26435 DoFs, and the H-formulation has 4111 vertices and 40950.

The benchmark test is carried out with respect to two different modes of criterion, one is P = 1.0 in average over the 10 HTS tapes (AVG), and the other is the same P but at maximum among those ones (MAX) [2]. As presented in the figures, the field profile of the H-field model just reproduces the reference model of A-formulation. In Table I, the result is summarized, in good agreements with the reference, arguing the potential of H-formulation as a practical alternative to the A-formulation.

I. SUMMARY OF THE BENCHMARK TEST

Background Field (T)	Formulation	I_c^{MAX} (A)	CPU time of I_c^{MAX} (s)	I_c^{AVG} (A)	CPU time of I_c^{AVG} (s)
0.0	H-formulation	535.64	14.77	539.05	15.89
	A-formulation	535.76	20.91	539.14	20.01
0.1	H-formulation	353.10	12.99	382.57	7.88
	A-formulation	352.88	27.11	382.34	34.05

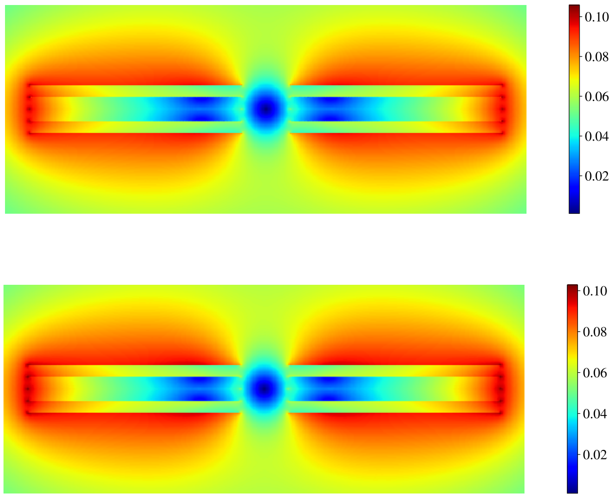
V. DISCUSSION IN PERSPECTIVES

Computing the vectorial field quantity directly from the discretized equation, the developed method has its own merits, for instance, the background field is imposed explicitly into the model, whereas the original formulation cannot help representing the background effect indirectly as far field vector potential on the infinite boundary. Moreover, the far field approximation itself becomes unnecessary for the boundary constraint, because the BEM part virtually evaluates the stray field in the outside, so that it is possible to reduce the redundancy of air mesh significantly. On those positive features, the technical considerations in plan are discussed as following;

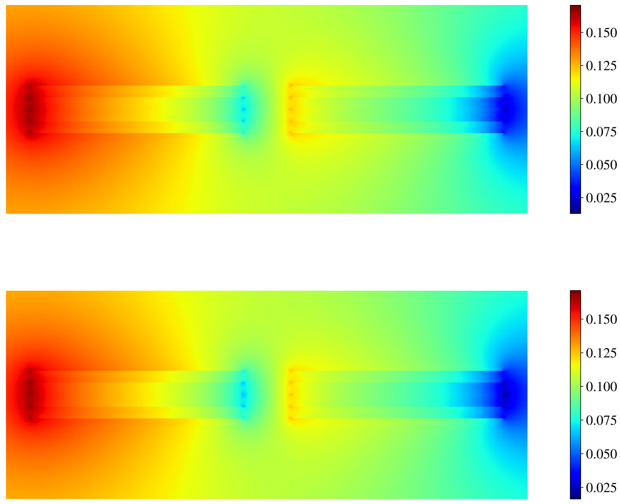
- A COMSOL complement of the same H-formulation is highly recommendable, taking into account its popular use in the HTS modeling community.
- Extending the developed method is also considerable, for instance, to the 3-d model, as well as to the hysteresis loss evaluation.
- Another means to impose the external field is feasible on the main domain of vectorial elements instead of the coupling integral along the boundary; it deserves to evaluate the cons and pros.

VI. CONCLUSION

The idea of H-formulation is successfully adapted to the outline of the previous method of critical current evaluation. In a straightforward manner, the developed mathematical formulations is translated to the numerical codes in the recentest FreeFEM++ platform. Through the benchmark result, the potential merits are discussed as a practical alternative to the established method for HTS modeling.



1. The critical current model of the Roebel cable without background field; the field maps are obtained from the H-formulation model (upper), and from the reference model in A-formulation (lower).



2. The critical current model of the Roebel cable with the background field of 0.1 T to y-axis; the field maps are obtained from the H-formulation model (upper), and from the reference model in A-formulation (lower).

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