Thermal Thin Shell Approximation for 3D Finite Element Quench Simulation of Insulated HTS Coils

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***Abstract*—Insulation layers in superconducting coils often exhibit high aspect ratios which can lead to unfavorable meshes, either because of a high number of degrees of freedom or deformed mesh elements. To avoid this issue, this work proposes to collapse thin insulation layers of insulated high-temperature superconducting coils into a surface using a thin shell approximation. This approach is beneficial for three-dimensional finite element method based quench simulations of coils made with high-temperature superconducting tapes.**

Keywords*—*thin shell approximation, quench simulation, HTS solenoid, finite element method, insulation layer

I. Introduction and Problem Definition

In order to study the quench behavior of superconducting coils, appropriate numerical tools are needed. When using the finite element (FE) method, the high aspect ratio of thin sheets such as insulation layers can lead to numerical difficulties [1] and suitable methods to deal with this problem are essential.

Let us consider a three-dimensional (3D) adiabatic high-temperature superconducting (HTS) solenoid $\Omega = \Omega_\mathrm{t} \cup \Omega_\mathrm{i}$, consisting of a homogenized HTS tape Ω_t and a thin insulation layer Ω_i (see Fig. 1). To analyze the thermal behavior, we solve the heat equation to find the temperature T such that

$$\nabla \cdot (\kappa \nabla T) + \rho C_\mathrm{p} \partial_t T = Q \qquad \text{in } \Omega, \qquad (1)$$

$$\vec{n}_\Omega \cdot (\kappa \nabla T) = 0 \qquad \text{on } \partial\Omega, \qquad (2)$$

where κ denotes the thermal conductivity, ρ the mass density, C_p the heat capacity, Q a heat source (e.g., given by Joule losses) and $\vec{n}_\Omega$ the unit vector normal to $\partial\Omega$.

By multiplying (1) with a test function T', integration over the computational domain and partial integration, the weak problem is to find $T \in \mathrm{H}^1(\Omega)$ such that

$$(\kappa \nabla T, \nabla T')_\Omega + (\rho C_\mathrm{p} \partial_t T, T')_\Omega = (Q, T')_\Omega \ \forall T' \in \mathrm{H}^1(\Omega),$$

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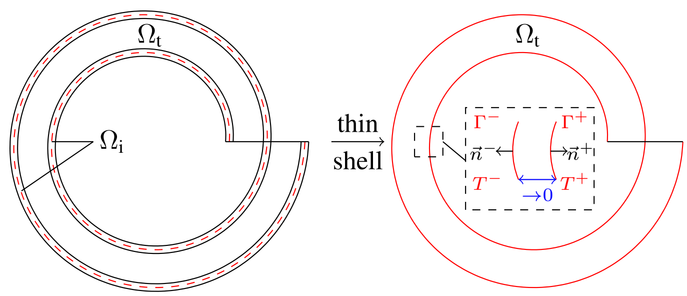

Fig. 1. Top view: in the proposed approach, the thin insulation layer in an HTS solenoid (left) marked by a dashed line is collapsed into a surface (right).

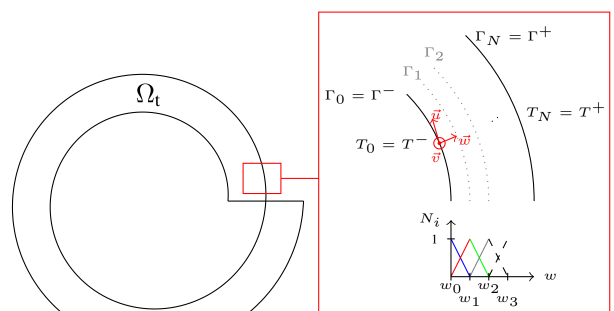

Fig. 2. Top view: virtual discretization of the insulation layer Ω_i into slabs.

where $\mathrm{H}^1(\Omega)$ is the space of square integrable functions with square integrable weak gradient in Ω and $(\vec{a}, \vec{a}')_\Omega$ denotes the volume integral of the scalar product of $\vec{a}$ and $\vec{a}'$ in Ω.

II. Thin Shell Approximation

The insulation layer Ω_i is collapsed into a surface Γ in the middle of the original layer (see Fig. 1) [2]. In order to model the temperature difference between the two sides of Γ, two copies Γ^+ and Γ^- of Γ at the same position allowing independent temperatures T^- and T^+ that represent the temperature on the two sides of the shell are introduced. As a consequence, the weak formulation now contains an additional surface integral, i.e.,

$$\begin{aligned}&(\kappa\,\nabla T,\nabla T')_{\Omega_\mathrm{t}}+(\rho\,C_\mathrm{p}\,\partial_t T,T')_{\Omega_\mathrm{t}}+\\ &\langle[\vec{n}\cdot\kappa\,\nabla T]\,,T'\rangle_\Gamma=(Q,T')_{\Omega_\mathrm{t}}\quad\forall T'\in\mathrm{H}^1(\Omega),\end{aligned}\tag{3}$$

where $\langle\vec{a},\vec{a}'\rangle_\Gamma$ denotes the surface integral of the scalar product of $\vec{a}$ and $\vec{a}'$ over Γ, $[b]:=b|_{\Gamma^+}-b|_{\Gamma^-}$ denotes the jump of b from Γ^- to Γ^+ and $\vec{n}=\vec{n}^+=-\vec{n}^-$ is the unit vector normal to the thin shell (see Fig. 2). Following the ideas proposed in [2] for the computation of magnetic fields using a thin shell approximation, the surface terms are now used for a *virtual discretization* of the heat equation in Ω_i which is split into N slabs $\hat{\Omega}_\mathrm{i}^{(k)}:=\Gamma\times[w_{k-1},w_k]$ (see Fig. 2) for $k=1,...,N$. Here, a local coordinate system $(\vec{u},\vec{v},\vec{w})$ is used with $(\vec{u},\vec{v})$ oriented along the tangential and $\vec{w}$ oriented along the normal direction of Γ. Then, we have

$$\begin{aligned}&\langle[\vec{n}\cdot\kappa\,\nabla T]\,,T'\rangle_\Gamma\\ &=(\kappa\,\nabla T,\nabla T')_{\hat{\Omega}_\mathrm{i}}+(\rho\,C_\mathrm{p}\,\partial_t T,T')_{\hat{\Omega}_\mathrm{i}}\\ &=\sum_{k=1}^{N}(\kappa\,\nabla T,\nabla T')_{\hat{\Omega}_\mathrm{i}}+\sum_{k=1}^{N}(\rho\,C_\mathrm{p}\,\partial_t T,T')_{\hat{\Omega}_\mathrm{i}}\,.\end{aligned}\tag{4}$$

For example, using first-order Lagrange basis functions

$$N_{k-1}(w)=\frac{w_k-w}{w_k-w_{k-1}}\quad\text{and}\quad N_k(w)=\frac{w-w_{k-1}}{w_k-w_{k-1}},$$

a one-dimensional discretization of T along the normal direction $\vec{w}$ is assumed as

$$T|_{\hat{\Omega}_\mathrm{i}^{(k)}}(u,v,w,t)=\sum_{j=k-1}^{k}T_j(u,v,t)N_j(w),$$

where $T_j:=T|_{\Gamma_j}$. Using the same ansatz for the test function T' subsequently leads to a decomposition of the last line of (4) into surface integrals over Γ_k and one-dimensional (1D) finite element matrices along w. For example, it follows that

$$(\rho\,C_\mathrm{p}\partial_t T,T')_{\hat{\Omega}_\mathrm{i}}=\sum_{k=1}^{N}\sum_{j=k-1}^{k}\langle\partial_t T_j,T_l'\rangle_{\Gamma_j}\cdot M_{lj,C_\mathrm{V}}^{(k)}\tag{5}$$

for $l=k-1,k$ and the 1D FE mass matrix

$$M_{lj,C_\mathrm{V}}^{(k)}:=\int_{w_{k-1}}^{w_k}\rho\,C_pN_l\,N_j\,\mathrm{d}w.\tag{6}$$

The remaining term in (4) yields similar results. These integrals can be evaluated without the need for a volumetric mesh representation of Ω_i while still allowing to consider multi-layered domains Ω_i by increasing the number of slabs N.

III. Numerical Experiments

The thin shell formulation has been implemented in the open-source FE framework GetDP [3] relying on Gmsh [4] for mesh creation. As before mentioned, this work proposes to use the latter formulation for efficient 3D FE quench simulations of HTS tape solenoids avoiding the cumbersome meshing of thin layers. For illustration, the heat distribution in a pancake coil resulting from the diffusion of a thermal hot spot leading to localized Joule heating is shown in Fig. 3. Good agreement between the results for a meshed insulation layer and the thin shell approach can be observed.

The proposed method will be compared to FE analysis with its counterpart with meshed insulation layers in terms of accuracy, efficiency and robustness. To enable comprehensive quench simulations, a weak coupling of the thermal thin shell approach and magnetic field computation will be discussed, in order to include Joule heating and the dependency of material properties (e.g., the critical temperature) on the magnetic field.

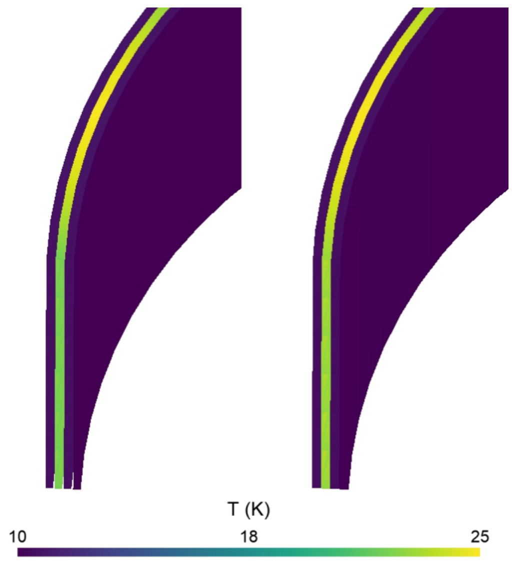

Fig. 3. 3D temperature distribution of diffusing hot spot in a small solenoid of four turns for meshed insulation layer (left) and thin shell insulation (right).

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