# Modelling high- $T_c$ superconducting dynamos

Mark Ainslie Department of Engineering University of Cambridge Cambridge, UK <u>mark.ainslie@eng.cam.ac.uk</u> ORCID: 0000-0003-0466-3680

Abstract—A high-T<sub>c</sub> superconducting (HTS) dynamo enables the injection of large DC currents into superconducting coil, without the need for thermally-inefficient current leads. Despite the extensive experimental work carried out over the past decade, there was - until very recently - some confusion and debate regarding the physical origin of the HTS dynamo's DC output voltage. Numerical modelling has played a key role in elucidating the underlying physics of such devices. Several different numerical models have now been developed as useful and cost-effective tools to not only explain and further examine experimental results but also optimise and improve dynamo designs. This review summarises all of the recent developments in this important area over recent years and provides a view towards the future, including the outstanding challenges and the developments required to address these.

Keywords—HTS dynamo, superconducting flux pump, high temperature superconductors, numerical simulation, HTS modelling

# I. INTRODUCTION

The high- $T_c$  superconducting (HTS) dynamo [1]–[3] exploits the nonlinear resistivity of an HTS coated-conductor wire to generate a DC voltage when subjected to a varying magnetic field. This effect enables the injection of large DC currents into a superconducting coil connected to the dynamo, without the need for thermally-inefficient current leads. Because of this important advantage, there is significant interest in using such technology to energise superconducting coils in superconducting rotating machines [4] and NMR/MRI magnets [5]. Despite the extensive experimental work carried out over the past decade, there has been some confusion and debate regarding the physical origin of the HTS dynamo's DC output voltage and quantitatively accurate, predictive calculations have been difficult to achieve. Numerical modelling has played a key role in elucidating the underlying physics of such devices. Several different numerical models have now been developed as useful and cost-effective tools to not only explain and further examine experimental results but also optimise and improve dynamo designs. This review summarises all of the developments in this important area over recent years and will provide a view towards the future, including the outstanding challenges and the developments required to address these.

## II. MODELLING OPEN-CIRCUIT VOLTAGE BEHAVIOUR

Mataira *et al.* [6] showed that the open-circuit voltage of the HTS dynamo can be explained well using classical electromagnetic theory. The 2D finite-element model in [6] implements the well-known H-formulation for the dynamo's

HTS stator wire, whose resistivity is described by the wellknown *E-J* power law, and the permanent magnet (PM) rotor is represented by a shell current which is rotated around the rotor boundary. Fig. 1 shows the open-circuit voltage waveforms presented in [6] using measured in-field  $J_c(B, \theta)$ data for the HTS wire or a constant  $J_c$  assumption, compared with experimental results, as well as the cumulative timeaverage for each waveform, which converges to  $V_{DC}$ , the DC output voltage, as  $t \rightarrow \infty$ . The DC output voltage of the dynamo arises naturally from a local rectification effect caused by overcritical eddy currents: an effect that has been observed in HTS materials as far back as Vysotsky *et al.* [7].



Fig. 1. (a) Open-circuit voltage waveforms for the HTS dynamo presented in [6] for the *H*-formulation + shell current model using measured in-field  $J_c(B, \theta)$  data for the HTS wire or a constant  $J_c$  assumption, compared with experimental results. (b) Cumulative time-average for each waveform, which converges to  $V_{\rm DC}$  in each case at  $t \rightarrow \infty$ .

#### A. A New Benchmark Problem

This work of Mataira et al. [6] generated significant interest amongst the modelling community to apply different frameworks to solve the problem and led to the definition of a new benchmark problem [8]. The geometry of the benchmark problem is shown in Fig. 2, assuming for simplicity the 2D case. A PM, of width a and height b, rotates anticlockwise past the stationary HTS stator wire at the top, and the face of the PM is located at a radius,  $R_{\rm rotor}$ . The initial position of the PM is such that the centre of its face is at  $(0, -R_{rotor})$ . The HTS wire has a width e and thickness f and is positioned such that its inner face is located at  $(0, R_{rotor} + airgap)$ .  $J_c$  is assumed to be constant – since it was shown in [6] that this assumption does not impact the essential dynamics to deliver a DC voltage, i.e., a non-linear resistivity via the E-J power law - and corresponds to  $I_{\rm c}$  [self-field, 77 K] = 283 A.

This benchmark was implemented using several different methods, including *H*-formulation-based methods, coupled H-A and T-A formulations, the Minimum Electromagnetic Entropy Production (MEMEP) method, and integral equation and volume integral equation-based equivalent circuit methods. Each of these approaches show excellent qualitative and quantitative agreement for the open-circuit equivalent instantaneous voltage - derived from the electric field averaged over the cross-section and the active length of the dynamo, corresponding to the active length (depth) of the PM - and the cumulative time-averaged equivalent voltage, as well as the current density and electric field distributions within the HTS wire at key positions during the PM transit. The benchmark has also been implemented successfully by Prigozhin and Sokolovsky [9] using expansions in Chebyshev polynomials for approximation in space and the method of lines for integration in time. Efficient 3D models [10], [11] have also been developed to take into account 3D considerations not possible with the 2D simplification.



Fig. 2. Geometry of the HTS dynamo benchmark problem [8]. A permanent magnet rotates anticlockwise past a (stationary) HTS wire.

### B. Investigating Key Dynamo Parameters

The presentation will also summarise other works investigating the key parameters of the HTS dynamo, including *V-I* characterisation for various frequencies [12], the gap dependence of the open-circuit voltage [13], the influence of the stator width [14], and the frequency dependence of the dynamo when taking into account the full HTS wire architecture coupled with a thermal model [15].

#### III. MODELLING DYNAMIC COIL CHARGING BEHAVIOUR

Of great interest from the perspective of practical applications is proper modelling of the dynamic behaviour of the dynamo while charging a coil. While the dynamo can be treated as a DC voltage source with an internal resistance [16], the dynamic current charging curve contains ripples within each cycle related to the PM transiting past the HTS wire. In [17], the authors use the MEMEP and segregated *H*-formulation methods to capture this behaviour, extending the benchmark problem by coupling the dynamo to a coil of inductance *L* with a circuit resistance,  $R_c$ , corresponding to the resistance of the soldered joints of the coil. This requires

redefining the voltage of the dynamo to include the vector potential contributions from the PM,  $A_{\rm M}$ , and the superconducting current induced in the HTS wire,  $A_{\rm J}$ . Fig. 3 shows a comparison of the output voltage when taking these two contributions into account with the equivalent instantaneous voltage defined in the benchmark problem. Fig. 4 then shows the dynamic charging current curve of the modelled coil for the two numerical methods, compared with analytical results.



**— —**  $\mathbf{E}_{av}$  **SEG-H — —**  $\mathbf{E}_{av}$  **+**  $\mathbf{A}_{J}$  **SEG-H —**  $\mathbf{E}_{av}$  **+**  $\mathbf{A}_{J}$  **+**  $\mathbf{A}_{M}$  **SEG-H** Fig. 3. Comparison of the three voltage components of the output voltage:  $l \cdot E_{av}$  and the two contributions to the vector potential A from the current density in the HTS wire,  $A_{J}$ , and permanent magnet,  $A_{M}$  [17].



Fig. 4. Dynamic charging current curve of the modelled coil in [17] for two numerical methods, MEMEP and the segregated *H*-formulation, compared with analytical results.

## IV. CONCLUSION & VIEW TOWARDS THE FUTURE

Several different numerical models of the HTS dynamo have now been developed as useful and cost-effective tools to not only explain and further examine experimental results but also optimise and improve dynamo designs. This review summarises all of the developments in this important area over recent years, including modelling the open-circuit voltage behaviour, the definition of a new benchmark problem for the HTS modelling community, investigating key dynamo parameters and modelling dynamic coil charging behaviour. A view towards the future will also be provided, including the outstanding challenges and the developments required to address these.

#### References

- C. Hoffmann, D. Pooke, and A. D. Caplin, "Flux Pump for HTS magnets," *IEEE Trans. Appl. Supercond.*, vol. 21, no. 3, pp. 1628– 1631, Jun. 2011.
- [2] J.Volger and P. S. Admiraal, "A dynamo for generating a persistent current in a superconducting circuit," *Phys. Lett.*, vol. 2, no. 5, pp. 257–259, 1962.
- [3] H.V. Beelen *et al.*, "Flux pumps and superconducting solenoids," *Physica*, vol. 31, no. 4, pp. 413–443, 1965.
- [4] C. W. Bumby *et al.*, "Development of a brushless HTS exciter for a 10 kW HTS synchronous generator," *Supercond. Sci. Technol.*, vol. 29, no. 2, 2016, Art. no. 024008.
- [5] Z. Bai *et al.*, "A novel high temperature superconducting magnetic flux pump for MRI magnets," *Cryogenics*, vol. 50, no. 10, pp. 688– 692, 2010.
- [6] R. C. Mataira *et al.*, "Origin of the DC output voltage from a high-T<sub>c</sub> superconducting dynamo," *Appl. Phys. Lett.*, vol. 114, no. 16, Art. no. 162601, 2019.
- [7] V. S. Vysotsky *et al.*, "The possibility of using high-Tc superconducting films as elements of a rectifier," *Supercond. Sci. Technol.*, vol. 3, no. 5, pp. 259-262, 1990.
- [8] M. D. Ainslie *et al.*, "A new benchmark problem for electromagnetic modelling of superconductors: the high-T<sub>c</sub> superconducting dynamo," *Supercond. Sci. Technol.*, vol. 33, no. 10, 2020, Art no. 105009.
- [9] L. Prigozhin and V. Sokolovsky, "Fast solution of the superconducting dynamo benchmark," *Supercond. Sci. Technol.*, vol. 34, no. 6, 2021, Art. no. 065006.
- [10] A. Ghabeli, E. Pardo, and M. Kapolka, "3D modeling of a superconducting dynamo-type flux pump," *Sci. Rep.*, vol. 11, 2021, Art. no. 10296.
- [11] L. Prigozhin and V. Sokolovsky, "Two-Dimensional Model of a High-T<sub>c</sub> Superconducting Dynamo," *IEEE Trans. Appl. Supercond.*, vol. 31, no. 3, Apr. 2021, Art. no. 5201107.
- [12] R. Mataira *et al.*, "Mechanism of the high-T<sub>c</sub> superconducting dynamo: Models and experiment," *Phys. Rev. Appl.*, vol. 14, no. 2, 2020, Art. no. 024012.
- [13] A. Ghabeli and E. Pardo, "Modeling of airgap influence on DC voltage generation in a dynamo-type flux pump," Supercond. Sci. Technol., vol. 33, no. 3, 2020, Art no. 035008.
- [14] R. Mataira *et al.*, "Modeling of stator versus magnet width effects in high-T<sub>c</sub> superconducting dynamos," *IEEE Trans. Appl. Supercond.*, vol. 30, no. 4, Jun. 2020, Art. no. 5204406.
- [15] M.D. Ainslie *et al.*, "Modelling the Frequency Dependence of the Open-Circuit Voltage of a High-T<sub>c</sub> Superconducting Dynamo," *IEEE Trans. Appl. Supercond.*, vol. 31, no. 5, Aug. 2021, Art. no. 4900407.
- [16] A. Pantoja *et al.*, "Output During Continuous Frequency Ramping of a Dynamo-Type HTS Flux Pump," *IEEE Trans. Appl. Supercond.*, vol. 28, no. 3, Apr. 2018, Art. no. 5202205.
- [17] A. Ghabeli *et al.*, "Modeling the charging process of a coil by an HTS dynamo-type flux pump," *Supercond. Sci. Technol.*, vol. 34, no. 8, 2021, Art. no. 084002.